

FP1 - May/June 2010

$$(1) \quad y_{n+1} = y_n + h \beta(x_n)$$

$$y_1 = 3$$

$$x_1 = 1$$

$$h = 0.1$$

$$\beta(x) = 1 + x^3$$

$$y_2 = 3 + 0.1 [1 + 1^3] \\ = 3.2$$

$$y_2 = 3.2$$

$$x_2 = 1.1$$

$$\beta(x) = 1 + 1.1^3$$

$$y_3 = 3.2 + 0.1 [1 + 1.1^3] \\ = 3.4331$$

$$y_3 = 3.4331$$

$$x_3 = 1.2$$

$$\beta(x) = 1 + 1.2^3$$

$$y_4 = 3.4331 + 0.1 [1 + 1.2^3] \\ = \boxed{3.7059}$$

$$(2) \quad a) \quad (1 - 2i)(x + iy) - (x - iy)$$

$$\rightarrow x + iy - 2xi + 2y - x + iy$$

$$\rightarrow 2y + 2iy - 2xi$$

$$\boxed{\text{REAL}} \quad 2y$$

$$\boxed{\text{IMAG}} \quad 2iy - 2xi \quad \text{or} \quad 2y - 2x$$

$$b) \quad \boxed{\text{REAL}} \quad 2y = 20 \quad \rightarrow \quad y = 10$$

$$\boxed{\text{IMAG}} \quad 2y - 2x = 10 \quad \rightarrow \quad x = 5$$

$$\rightarrow \quad z = 5 + 10i$$

③ $\theta = 360n \pm a$

Key angle = $\cos^{-1}(\cos 40) = 40$

$\rightarrow 5x - 20 = 360n \pm 40$

$5x = 360n + 20 \pm 40$

$x = 72n + 4 \pm 8$

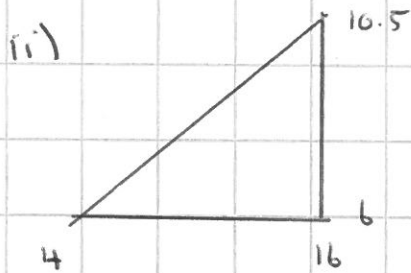
or $72n + 12, 72n - 4$

④ a)

x	2	4	6	8
x	4	16	36	64

b) see mark scheme

c) i) $y = 15, x = 28 \rightarrow x^2 = 28 \rightarrow x = \sqrt{28} = 5.3$

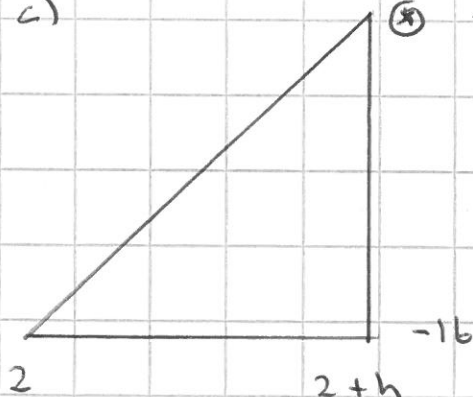


Gradient = $\frac{4.5}{12} = 0.375$

y intercept ≈ 4.5

$\rightarrow y = 0.38x^2 + 4.5$

⑤ c)



$\rightarrow (*) (2+h)^3 - 12(2+h)$
 $= 2^3 + 3 \times 2^2 \times h + 3 \times 2 \times h^2 + h^3$
 $- (24 + 12h)$
 $= 8 + 12h + 6h^2 + h^3 - 24 - 12h$
 $= h^3 + 6h^2 - 16$

$$\begin{aligned} \text{Gradient} &= \frac{h^3 + 6h^2 - 16}{2+h-2} - (-16) \\ &= h^2 + 6h \end{aligned}$$

b) As $h \rightarrow 0$, gradient $\rightarrow 0$

\therefore Gradient at A = 0, which means it's a stationary point.

(b) a) $\cos \theta = 1/\sqrt{2} \rightarrow \theta = 45^\circ$

Rotation, 45° , Anti-clockwise

b) $\cos 2\theta = 1/\sqrt{2} \rightarrow \theta = 22.5^\circ$

Reflection in line $y = (\tan 22.5)x$

c) $\begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$

$\begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$\cos \theta = 0$

$\rightarrow \theta = 90$

Rotation 90° anti-clockwise.

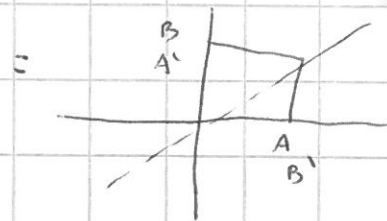
[NOTE: I should have known this by looking at A!]

d) B^2 must be a reflection followed by same reflection

$\rightarrow I$ or $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

e)

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

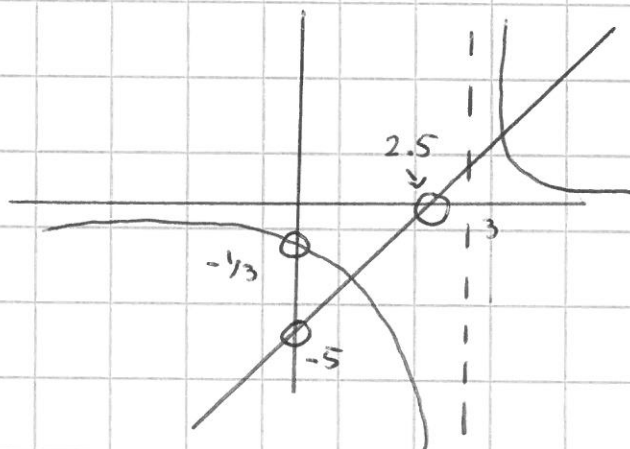


= Reflection in line $y=x$

⑦ a) i) $\boxed{xc=3}$ As $x \rightarrow \infty, y \rightarrow \frac{1}{\infty} = 0$
 $\rightarrow \boxed{y=0}$

ii)

iii)



b) i) $1 = \frac{2x-5}{x-3}$

$$\rightarrow 1 = (2x-5)(x-3)$$

$$1 = 2x^2 - 6x - 5x + 15$$

$$1 = 2x^2 - 11x + 15$$

$$2x^2 - 11x + 14 = 0$$

$$(2x-7)(x-2) = 0$$

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$$x = \frac{7}{2}$$

$$x = 2$$

ii) Curve lies below line: $2 < x < 3$ and $x > \frac{7}{2}$

8) a) $\alpha + \beta = 4$, $\alpha\beta = 10$

b) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta}{\alpha\beta} + \frac{\alpha}{\alpha\beta} = \frac{\beta + \alpha}{\alpha\beta}$

$= \frac{4}{10} = \frac{2}{5}$

c) Sum $\alpha + \frac{2}{\beta} + \beta + \frac{2}{\alpha} = \alpha + \beta + \frac{2\alpha}{\alpha\beta} + \frac{2\beta}{\alpha\beta}$
 $= \alpha + \beta + \frac{2(\alpha + \beta)}{\alpha\beta}$

$= 4 + \frac{8}{10}$
 $= 2\frac{4}{5}$

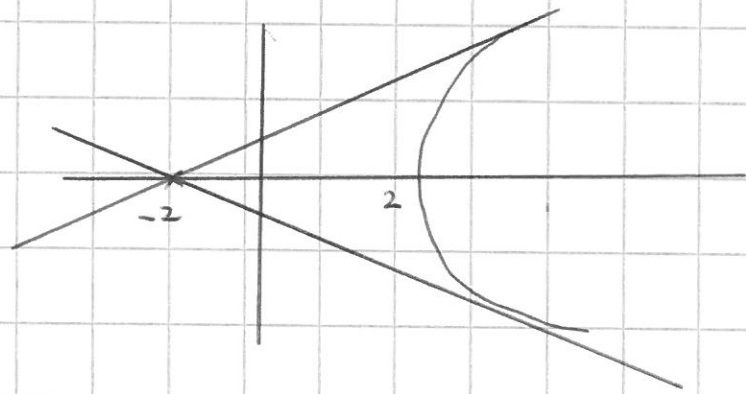
Product $(\alpha + \frac{2}{\beta})(\beta + \frac{2}{\alpha})$
 $= \alpha\beta + 2 + 2 + \frac{4}{\alpha\beta}$
 $= 10 + 4 + \frac{4}{10} = 7\frac{2}{5}$

$\rightarrow x^2 - \text{[Sum]}x + \text{[Product]} = 0$

$\rightarrow x^2 - 2\frac{4}{5}x + 7\frac{2}{5} = 0$

$\rightarrow 5x^2 - 24x + 72 = 0$

9) i)
ii)



b) i) $y = m(x+2)$

At intersection:

$m^2(x+2)^2 = x-2$

$m^2[x^2 + 4x + 4] = x-2$

$$m^2 x^2 + 4m^2 x + 4m^2 - x + 2 = 0$$

$$(m^2)x^2 + (4m^2 - 1)x + (4m^2 + 2) = 0$$

ii) For equal roots, $b^2 - 4ac = 0$

$$\rightarrow (4m^2 - 1)^2 - 4 \times m^2 \times (4m^2 + 2) = 0$$

$$\rightarrow 16m^4 - 8m^2 + 1 - 16m^4 - 8m^2 = 0$$

$$\rightarrow -16m^2 + 1 = 0$$

$$\rightarrow 16m^2 = 1$$

iii) $m^2 = 1/16$

use: $(m^2)x^2 + (4m^2 - 1)x + (4m^2 + 2) = 0$

$$\rightarrow \frac{1}{16}x^2 + \left(\frac{4}{16} - 1\right)x + \frac{4}{16} + 2 = 0$$

$$\rightarrow \frac{1}{16}x^2 - \frac{3}{4}x + \frac{9}{4} = 0$$

$$\rightarrow x^2 - 12x + 36 = 0$$

$$(x - 6)(x - 6) = 0$$

$$\rightarrow x = 6$$

Find y:

$$y^2 = x - 2$$

$$y^2 = 6 - 2$$

$$y^2 = 4$$

$$\rightarrow y = \pm 2$$

$$\therefore (6, -2) \text{ and } (6, 2)$$